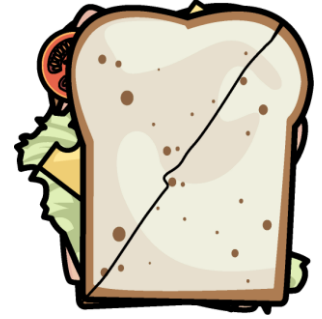


SQUEEZE



Theorem

NOTE: (also called the Sandwich Theorem OR Pinching Theorem)

If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a)
& $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$,

then $\lim_{x \rightarrow a} g(x) = L$.

Examples: 1. Show that, $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0$

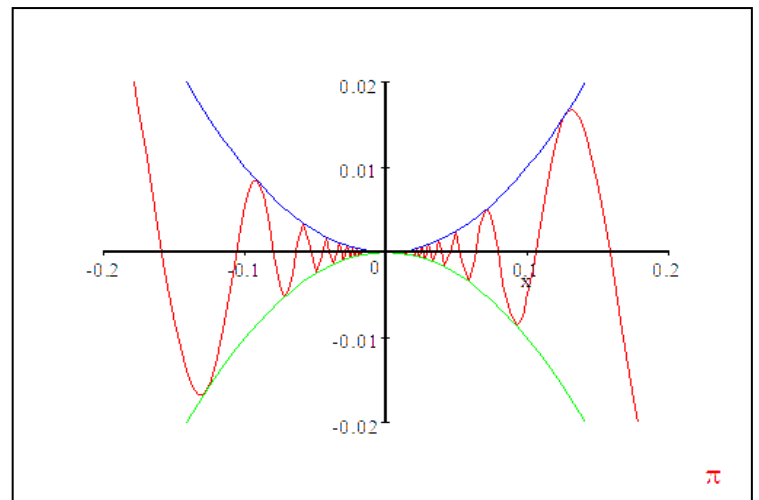
Can't use $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0$ because the $\sin \frac{1}{x}$ does not exist as $x \rightarrow 0$.

We know $-1 \leq \sin \frac{1}{x} \leq 1$

So, $-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$

$$\lim_{x \rightarrow 0} (-x^2) = 0 \quad \lim_{x \rightarrow 0} x^2 = 0$$

$$\therefore \lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0$$



Apply the Squeeze Theorem to the following to determine the limit.

2. If $2x + 2 \leq f(x) \leq x^2 + 3$
for all values of $0 \leq x \leq 2$

then $\lim_{x \rightarrow 1} f(x) =$

3. If $-x^2 - 2 \leq g(x) \leq 2x - 1$
for $-2 \leq x \leq 0$

then $\lim_{x \rightarrow -1} g(x) =$